Hypervolume in Biobjective Optimization Cannot Converge Faster Than $\Omega(1/p)$

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Biographie – I have an engineering degree from ENSTA ParisTech and a M2 in Optimization from Paris Saclay. I started my PhD in 2019. I have a founding from Ecole Polytechnique and work at CMAP in the INRIA team Randopt. I do not focus on specific applications, but multiobjective optimization algorithms have many applications in industry (design of cars, radars, ...). I analyze such algorithms from a mathematical perspective, in particular their convergence properties.

Resumé : I intend to present the content of [1], which has been accepted at the GECCO Conference 2021. In my talk, I would first introduce the domain of multiobjective optimization and how it differs from the more famous singleobjective optimization. Then, I will present the main result from this paper, i.e that when measuring convergence to the Pareto front (the set we look for in multiobjective optimization) with the hypervolume indicator (a widely used set-quality indicator), no algorithm can converge faster than $\Omega(1/p)$ on a wide class of problems, p being the number of function evaluations. I will present the key idea of the proof and some numerical experiments on real Pareto fronts.

The abstract of [1] is copied below.

The hypervolume indicator is widely used by multi-objective optimization algorithms and for assessing their performance. We investigate a set of p vectors in the biobjective space that maximizes the hypervolume indicator with respect to some reference point, referred to as *p-optimal distribution*. We prove explicit lower and upper bounds on the gap between the hypervolumes of the p-optimal distribution and the ∞-optimal distribution (the Pareto front) as a function of p, of the reference point, and of some Lipschitz constants. On a wide class of functions, this optimality gap can not be smaller than $\Omega(1/p)$, thereby establishing a bound on the optimal convergence speed of any algorithm. For functions with either bilipschitz or convex Pareto fronts, we also establish an upper bound and the gap is hence $\Theta(1/p)$. The presented bounds are not only asymptotic. In particular, functions with a linear Pareto front have the normalized exact gap of $1/(p+1)$ for any reference point dominating the nadir point.

We empirically investigate on a small set of Pareto fronts the exact optimality gap for values of p up to 1000 and find in all cases a dependency resembling $1/(p + \mathrm{CONST})$.

Références

[1] Eugénie Marescaux and Nikolaus Hansen. Hypervolume in Biobjective Optimization Cannot Converge Faster Than $\Omega(1/p)$. In *GECCO 2021 - The Genetic and Evolutionary Computation Conference*, Lille / Virtual, France, July 2021.