# A symmetric algorithm for solving mechanical contact problems using FreeFEM

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**Biographie** – Houssam Houssein doctoral candidate in third year at Sorbonne Université, I am in a CIFRE thesis between the laboratory Laboratoire Jacques-Louis Lions (LJLL) and the startup Airthium. This thesis was created following a need to develop an algorithm on FreeFEM, to solve mechanical contact problems with some features. Finally what will allow the company to use it to study their energy storage device.

**Resumé :**

The mechanical contact between two bodies is one of the most difficult problems in solid mechanics, indeed the material non-linearity must be taken into account and the contact area is unknown. In the case of frictional contact, another non-linearity must be considered and makes the problem even more difficult. There exist several algorithms to solve the contact problems [6], most of them involve the concept of master/slave, which prevents the penetration of the slave body into the master one, and therefore causes the non-symmetry of the algorithm.

In the first part of this work, the friction is not taken into account, and an algorithm solving the contact between two bodies using FreeFEM [1] is presented. The contact problem is formulated into a constrained minimization one, on one hand the constraints for linear elastic problems, which describe the non-penetration between the two bodies, are linear. On the other hand the constraints are non-linear for large deformation problems, and therefore a fixed point algorithm is employed in order to transform the problem into a sequence of minimization ones with linear constraints, and the resolution is done using the interior-point method [2]. One of the advantages of this algorithm is the symmetric behavior, in order to allow the user to no longer distinguish between slave and master bodies. Moreover the Interior point optimizer (IPOPT) [5] is used in order to solve the constrained minimization problem.

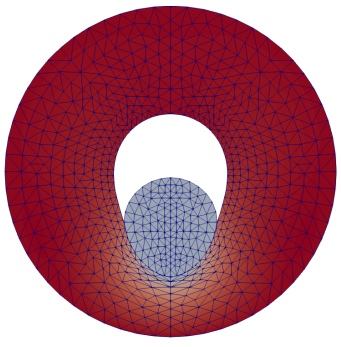

Figure 1: Contact between two discs

In the second part of this work, the frictional contact problem is considered. There exist several friction laws to describe the friction, the widely used one is Coulomb's law. Unfortunately the contact problem using Coulomb's law cannot be written into a minimization one, thus we cannot use and modify our first contact algorithm which is based on a minimization principle. In order

to overcome this issue, the frictional contact problem using Coulomb's law will be written as a sequence of contact problems (more specifically as a fixed point algorithm [4]) using Tresca's law, which is equivalent to a minimization problem. For a each problem with Tresca's law, there is a non-differentiable term, therefore we will regularize this term [3, 4] in order to have a sufficiently smooth problem, and to use the interior point method to solve the latter. Some theoretical results and validation examples will be presented.

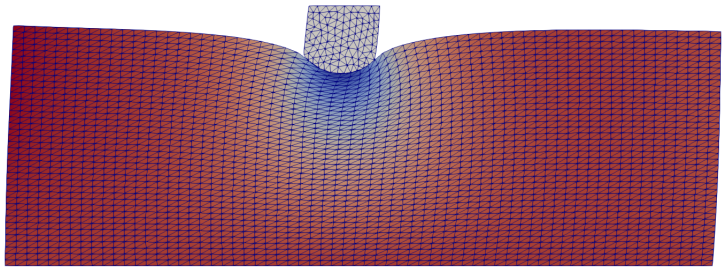

Figure 2: Shallow ironing problem with friction

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