Boundary stabilization of a one-dimensional cross-diffusion system in a moving domain using backstepping

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Biographie – Ancien étudiant de l'Ecole des Ponts et Chaussées, j'ai une formation générale en mathématiques appliquées avec un intérêt particulier pour l'analyse et les équations aux dérivées partielles. J'ai débuté mon projet thèse autour des "Systèmes de diffusion croisée dans des domaines mobiles" en septembre 2020, sous la direction de Virginie Ehrlacher au Cermics à l'Ecole des Ponts, avec une affiliation à l'équipe Inria MATHERIALS. Cette thèse est financée par le projet ANR Comodo de Virginie Ehrlacher.

Resumé :

We are interested in a particular one-dimensional cross-diffusion system that was proposed and studied in [1] to model the Physical Vapor Deposition (PVD) process used e.g for the fabrication of thin film crystalline solar cells. The procedure works as follows: a wafer is introduced in a hot chamber where several chemical elements are injected under a gaseous form. As the latter deposit on the substrate, a heterogeneous solid layer grows upon it. In the model, the solid layer is composed of $n+1$ different chemical species and occupies a domain of the form $(0, e(t)) \subset \mathbb{R}_+$, where $e(t) > 0$ denotes the thickness of the film and is determined by the fluxes ϕ_i of atoms that are absorbed at the surface of the layer:

$$e(t) = e_0 + \int_0^t \sum_{i=0}^{n} \phi_i(s) ds,$$

The cross-diffusion equation in the bulk, together with the flux boundary conditions, form the system:

$$\begin{cases} \partial_t u - \partial_x(A(u)\partial_x u) = 0, & \text{for } t \in \mathbb{R}_+^*,\ x \in (0, e(t)), \\ (A(u)\partial_x u)(t, 0) = 0, & \text{for } t \in \mathbb{R}_+^*, \\ (A(u)\partial_x u)(t, e(t)) + e'(t)u(t, e(t)) = \phi(t), & \text{for } t \in \mathbb{R}_+^*, \end{cases} \tag{1}$$

where $A(u)$ is the *diffusion matrix* of the system encoding the cross-diffusion properties of the different species. It has an explicit expression, but is neither symmetric nor positive definite, which makes the analysis of such systems challenging.

In [1] global existence of weak solutions was proved, adapting the *boundedness-by-entropy method* ([3]). The authors also proved long-time asymptotics in the case of constant external fluxes: the solution converges (in a *rescaled* L^1 sense) to constant stationnary states with rate at least $\mathcal{O}(\frac{1}{\sqrt{t}})$.

In this communication, we are interested in achieving better rates of convergence (e.g exponential or finite time) to stationnary states by controlling the external gas fluxes ϕ_i. I will give an introduction to the analysis of cross-diffusion systems and present the results obtained for the linearized system, following the *backstepping* approach of [2]. This is a joint work with Virginie Ehrlacher and Amaury Hayat at Ecole des Ponts.

Ongoing work is concerned with the treatment of the nonlinear terms in (1). In the future, we would like to propose and study a higher dimensional model for PVD that would include *surfacing diffusion effects*.

Références

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