Capillarity-Shear Equilibrium between Two Immiscible Stokes Fluids

S. Piccardo[1,2,3], A. Ern[1,2]
[1]*CERMICS, École des Ponts;* [2]*INRIA Paris;* [3]*LaCàn, Universitat Politecnica de Catalunya,*

Email : stefano.piccardo@enpc.fr

Mots Clés : Hybrid discretization methods, Finite element methods, Stokes flows, Immiscible fluids, Surface tension

Biographie – My PhD thesis is a cotutelle between CERMICS laboratory, École des Ponts (ENPC), and LaCàn laboratory, Universitat Politecnica de Catalunya (UPC), under the supervision of A. Ern, A. Huerta and M. Giacomini. The Thesis started in the Fall 2019. The first 18 months have been spent at CERMICS, and the second part is now starting at UPC.

Resumé : This presentation deals with the equilibrium of two immiscible, incompressible Stokes fluids separated by an interface, where surface tension effects are present. The interface splits the computational domain into two subdomains, and each subdomain is occupied by a fluid governed by a steady, incompressible Stokes equation. At the interface, the fluid velocities are continuous, whereas the jump of the normal total stress is proportional to the curvature of the interface. These equations define the so-called Stokes interface problem. Instead, in the interface equilibrium problem, the Stokes interface problem has to be completed by requiring that the normal velocity at the interface is zero. This last condition enforces the equilibrium and essentially prescribes the shape of the interface.

Let us first consider the Stokes interface problem with prescribed interface, but without the equilibrium condition. In this case, any interface can be considered. Notice that the Stokes interface problem is well-posed. To approximate its solution, we consider an unfitted hybrid high-order (HHO) method. In a nutshell, the method is called unfitted because it employs a mesh that does not fit the interface, that is, the interface can cut arbitrarily through some of the mesh cells. The jump conditions at the interface are then enforced by means of a consistent penalty technique inspired by Nitsche's method to weakly enforce non-homogeneous Dirichlet conditions in elliptic problems. Moreover, the space differential operators are discretized using the HHO method. This method employs hybrid unknowns (face- and cell-based) for the velocity and cell-based unknowns for the pressure. Among its assets, for incompressible Stokes flows, the HHO method is inf-sup stable, locally conservative, and supports polytopal meshes. Moreover, the HHO method is computationally efficient, owing to its compact stencil and thanks to the possibility of a local elimination of the cell velocity unknowns by a static condensation procedure.

HHO methods on fitted meshes have been introduced in [3] for locking-free linear elasticity. Unfitted HHO methods for elliptic interface problems have been studied in [1], where a local cell-agglomeration procedure is devised to counter the adverse effects of unfavorably cut cells. Unfitted HHO methods for the Stokes interface problem have been analyzed in [2], which constitutes the starting point of the present work. Herein, we extend this work in two directions. First, [2] does not consider surface tension effects, whereas we do. This, in particular, requires us to approximate the curvature of the interface at all the integration points along the interface. Second, the quadratures in cut cells used in [1] are based on a sub-partition of the cut cell using affine triangles (in a quite large number to accurately approximate the curved interface), whereas, for the quadratures, we use an isoparametric description of the interface requiring much fewer curved triangles.

In a second step, we study the interface equilibrium problem in the particular case where there are no body forces and a shear flow is prescribed at infinity. In this case, the equilibrium interface

is an ellipse, and the ratio of its two diameters depends upon the ratio of the prescribed shear to surface tension [5]. When the ratio of shear to surface tension is zero (no prescribed shear), the ratio of the two diameters is unity, that is, the ellipse becomes a circle. We investigate numerically the dependence of the shape of the equilibrium ellipse on the shear to surface tension ratio and find a linear relationship. To this purpose, we pose the Stokes interface problem on a finite computational box surrounding the elliptic equilibrium interface, and observe numerically that the expected equilibrium is fairly well attained even on moderately large computational domains.

In the last part, we consider the more challenging objective in which the boundary conditions (and possibly body forces) are more complex, so that the equilibrium interface no longer has an elliptic shape. The shape of the interface is thus an unknown of the problem that has to be determined by requiring that the normal velocity at the interface vanishes. We devise a fixed-point iterative procedure aiming at approaching the equilibrium interface by a successive resolution of Stokes interface problems and a transport of the level-set function describing the interface by the flow field resulting from the Stokes interface problem. To transport the level-set function, we use a finite element method with graph-viscosity recently devised [4] for that purpose (the method can be enhanced by an entropy-viscosity and flux correction limiters).

Références

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